301. Visualise the possibility space with a tree diagram. You don't need to draw the whole tree, you just need to keep in mind its structure, particularly how non-replacement affects the fractions.

302. Consider $x = 0$.

303. Square both sides, using $|a| = |b| \implies a^2 = b^2$.

304. Consider Δ or equivalently double roots.

305. Substitute $(0, 0)$.

306. Consider the coordinates of the vertex.

307. This is an ambiguous case of the sine rule: there are two possible answers.

308. (a) Use the discriminant $\Delta = b^2 - 4ac$.
   (b) Sketch $y = x^2 + 1$ and $y = x$, and consider the y values of the former compared to the y values of the latter.

309. Substitute the values to set up a pair of equations in a and b. Solve these.

310. Set the output of the function equal to a variable y. Rearrange this to make x the subject. Then, rewrite the inverse function with x as its input.

311. Use the factor theorem and the discriminant.

312. Whether you use the substitution or not, factorise a quadratic (in either z or x^3).

313. Division by x^2 is undefined if $x^2 = 0$.

314. Differentiate to find $\mathrm{f}'(x)$ and substitute.

315. Prove that the line segment joining $(5, 0)$ to $(4, 2)$ is perpendicular to the hypotenuse of the larger right-angled triangle.

316. The relevant definite integral is

$$\int t - 1\, dt = \tfrac{1}{2}t^2 - t + c.$$

Evaluate to get a quadratic in k, and factorise.

317. Write x in terms of y and substitute.

318. Consider NII for equilibrium, and NIII.

319. (a) Use the factor theorem.
   (b) The graph is a pair of concentric circles, with radii 1 and 2.

320. Consider the area scale factor.

321. Take out a common factor.

322. (a) The bar means "..., evaluated at ..."
   (b) Use the factor theorem.

323. If in doubt, sketch a number line.

324. Remember that an integral calculates the *signed* area. This is to say that negative-valued functions produce negative-valued signed areas.

325. Multiply the number of paths on the way out by the number of available paths on the way back.

326. When fractions are inlaid like they are on the LHS, multiply top and bottom of the large fraction by the denominator of the little fraction(s).

327. The key word is "light".

328. Consider the scale factor required to convert 80 marks into 100 marks.

329. Use the binomial expansion on the LHS, including the three ingredients:
   1. binomial coefficients from Pascal's triangle,
   2. descending powers of 1,
   3. ascending powers of $\sqrt{3}$.

   Note that e.g. $\left(\sqrt{3}\right)^3$ can be simplified as $3\sqrt{3}$.

330. There are two isosceles triangles, one along the top edge and one along the bottom. Find the area of these, and the area of the equilateral triangles. The shaded region is "two equilateral triangles plus two isosceles triangles minus a square."

331. Take a square root, remembering a $\pm$.

332. You don't need to find angles. The magnitudes $(6, 8, 10)$ form a Pythagorean triple, twice $(3, 4, 5)$. Its sine/cosine ratios are $3/5$ and $4/5$.

333. Consider standard polynomial integration.

334. (a) The centre is $(1, 1)$.
   (b) To stretch by scale factor 2 in the x direction, replace x by $x/2$.
   (c) Switch x and y.

335. Consider the mean of the sequence.

336. (a) Calculate the squared distance between each pair of adjacent vertices.

(b) The area of a rhombus is most easily calculated from the lengths of its diagonals.

337. Complete the square or differentiate.

338. "Pairwise mutually exclusive" means that no two of the events overlap.

339. Use $S_n = \frac{1}{2}n\big(2a + (n-1)d\big)$.

340. Consider the fact that $2 + 5 < 8$.

341. The notation here, which is the standard notation in which to express definite integration, has the following meaning:

$$\Big[\mathrm{F}(x)\Big]_a^b = \mathrm{F}(b) - \mathrm{F}(a).$$

342. Draw a sketch, but don't use <u>coordinate</u> geometry. Instead, use the fact that tangent and radius are perpendicular, and Pythagoras.

343. You don't need any calculations here; just use a symmetry argument.

344. (a) Integrate the second derivative, including a $+c$. Then substitute $x = 0$.
  (b) Use the same technique as in part (a), now with the first derivative.
  (c) The curve is a positive parabola. Check Δ to see whether it crosses the x axis.

345. Use the factor theorem to show that $(x + 1)$ and $(x - 1)$ are both factors.

346. Divide both sides of $l = r\theta$ by $2\pi r$. Hence, prove that $\frac{\theta}{2\pi}$ is the fraction of the circumference/area of the circle occupied by the arc/sector. Multiply πr^2 by this scale factor.

347. Set up a right-angled triangle of forces, arranging the vectors tip-to-tail. Then use Pythagoras.

348. Multiply out the brackets in the numerator, then split up the fraction. Integrate term by term.

349. Use a conditioning argument, visualising dealing the cards one by one. The possibility space would be a tree diagram, although it's too big to draw. There are 40 non-face cards in a pack.

350. (a) Complete the square.
  (b) Consider the fact that a squared bracket has a minimum value of 0.

351. Functions with constant derivatives are linear. You can either count explicit steps of 3, or use calculus, or use $y = mx + c$.

352. The factor $(x - a)$ changes sign at $x = a$. What about powers of the same factor?

353. Multiply out and rearrange to a linear equation in $\sqrt{x}$.

354. (a) Use the cosine rule in the form

$$\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}.$$

  (b) Use $\sin^2\theta + \cos^2\theta = 1$, explaining why you take the positive square root.
  (c) Use the value from (b).

355. The next step is $y^2 - 2 = \pm 5$.

356. Part (b) involves an overcounting argument: there are half as many arrangements in (b) as (a).

357. Integrate and find the $+c$.

358. Despite the unknown constants, this is a standard quadratic and can be factorised as usual.

359. Consider the possibility $x = -y$.

360. One revolution is a full circle.

361. Consider the gradients of the lines.

362. Find the area of the eight small triangles around the outside. Subtract the area of four of them from the area of a square.

363. Use Pythagoras.

364. Sketch regions on a graph.

365. The space diagonal is the longest diagonal. Use 3D Pythagoras: $d = \sqrt{x^2 + y^2 + z^2}$.

366. Find the magnitude of the resultant force using a one-dimensional *suvat*, then find $\mathbf{F}_3$.

367. Write down the coordinates of the vertex i.e. the turning point of the parabola, and construct the new curve from there.

368. (a) Multiply out.
  (b) Make y the subject.

369. Firstly, solve the inequality without reference to the "integer" fact.

370. The interior of the square is the possibility space of locations for the centre of the circle. Locate the region inside which placement of the centre will produce intersection. Then use the formula $p = \frac{\text{successful}}{\text{total}}$, referring to area.

371. A polynomial identity makes multiple individual equations, one for each coefficient. These cannot necessarily be solved simultaneously. Here, you are looking for an explicit contradiction.

372. Use the factor theorem.

373. You can explain this in English. However, it is easier and more rigorous to use integration.

374. Draw in the square and find its area.

375. Call the odd numbers $2a+1$ and $2b+1$, for $a, b \in \mathbb{Z}$.

376. Find the minimum value of $y = 7x^2 - 9x + 12$, either by calculus or by completing the square.

377. Consider negative numbers.

378. Consider the boundary equation $xy = 0$.

379. (a) Find displacement with a definite integral.
     (b) Substitute the values of k and solve.

380. No calculation is needed here: integration is anti-differentiation. Just be careful with the constant.

381. Divide both sides by $\cos^2\theta$.

382. It's a minus sign.

383. Use calculus, then return to integers afterwards.

384. Consider the $\mathbf{i}$ and $\mathbf{j}$ directions separately. This will give you a pair of simultaneous equations in a and b.

385. $a \propto b$ means $a = kb$ for some constant k.

386. Use the following result:

$$\int_a^b g'(x)\,dx = g(b) - g(a).$$

387. Compare the number of successful outcomes in the possibility space.

388. Both transformations are in the y direction.

389. Subtract the triangles from the rectangle.

390. Rearrange the first equation to make y the subject, and substitute it into the second equation.

391. Since two of the vertices of the quadrilateral are on the x axis, it can be split up into three parts: two triangles and a trapezium.

392. Set up and solve a pair of simultaneous equations by comparing coefficients of $\mathbf{i}$ and $\mathbf{j}$.

393. Since the t values form an AP, you can treat the x values as a sequence. Find its second differences.

394. (a) Solve using the quadratic formula.
     (b) Use the factor theorem.

395. List alphabetically.

396. Consider the average value of the angles.

397. (a) In 2D, non-parallel lines always intersect.
     (b) Compare gradients.
     (c) All but one value of q is possible.

398. Consider the shape of a parabola. The range is the set of all possible outputs.

399. Consider the parity (evenness or oddness) of n and $n+1$.

400. Consider an object floating in space.

——— End of 4th Hundred ———